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# Mass transfer on unsteady MHD couette flow of non-newtonian fluid in horizontal channel with chemical reaction effects

**Joseph Kpop Moses[1*], Tijjani Nasiru Zakari[2]**
[1] Department of Mathematical Sciences, Kaduna State University, Kaduna, Nigeria
[2] Department of Mathematics and Statistics, Kaduna Polytechnic, Kaduna, Nigeria

## Abstract
This paper analyzes the mass transfer on unsteady MHD flow of non-Newtonian fluid in horizontal channel with chemical reaction effects. The equations that govern the flow are transformed into non-dimensional form using dimensionless parameters. The non-dimentional equations are solved by implementing He-Laplace method. The influence of diversified flow parameters such as chemical reaction, Schmidt number, Grashof number due to mass transfer, Hartmann number are presented and discussed graphically on different flow fields. An interesting fact is that the velocity and concentration fields diminish due to the increment of chemical reaction parameter. For upsurging data of Schmidt number, velocity and concentration fields diminish. Furthermore, the velocity field decline due to the increment of magnetic parameter.

**Keywords:** MHD, mass transfer, chemical reaction, non-newtonian

**Introduction**
Chemical reaction is a process that involves rearrangement of the molecular or ionic structure of a substance, as distinct from a change in physical form or nuclear reaction. There are two types of such reactions namely homogeneous reaction which occurs uniformly throughout a given phase of a flow and heterogeneous reaction which takes place in a particular region or within the boundary of a phase Umavathi [1]. The study of heat transfer with chemical reaction is of most realistic significance to engineers and scientists because of its universal incidence in many branches of science and engineering. This phenomenon plays a significant role in chemical industry, power and cooling industry for dying, evaporation, energy transfer in cooling tower and flow in s desert cooler, etc. Satya et al [2].
A vast scientific analysis of non-Newtonian fluids problems has been done by many researchers. This has gained great importance in different fields due to their huge range of engineering and commercial applications. The study of the behavior of the motion of non-Newtonian fluids is very much more complicated and difficult as compared to that for Newtonian fluids, because of the nonlinear relationship between the stress and the rate of strains. The governing equations that describe the flow of Newtonian fluid is the Navier-Stokes equations, while for the flow of the non-Newtonian fluids there is no single governing equation which describes all of their properties and thus it is difficult to describe these fluids as Newtonian fluids. Therefore, many empirical and semi-empirical non- Newtonian models or constitutive equations have been proposed Islam et al. [3], where was considered the steady flow of a non-Newtonian fluid with slippage between the plate and the fluid. The constitutive equations of the fluids were modelled for fourth-grade non-Newtonian fluid with partial slip. They employed homotopy perturbation and optimal homotopy asymptotic methods to solve the non-linear differential equation.
Hayat et al. [4] examined the unsteady flow of a hydrodynamic fluid past a porous plate. The constitutive equations of the fluids were modelled by therefore a fourth-grade fluid. The study gave rise to a boundary value problem consisting of a fifth-order differential equation but there were only two boundary conditions. The solution was obtained by implementation of Lie group method. In another study, Hayat et al. [5] used Laplace transform method to determine the analytical solution of couette flows of a second grate fluid. Stokes and couette flows due to oscillating wall were discussed by Khaled and Vafai [6]. Singh [7] studied the periodic solution of oscillatory couette flow of through porous medium in rotating system. Guria [8] discussed couette flow problem for rotating and oscillatory flow.
Idowu et al. [9] considered the unsteady Couette flow with transpiration of a viscous fluid in a rotating system. An exact solution of the governing equations has been obtained by using Laplace Transform Technique.
Couette flow of an unsteady third grade fluid with variable magnetic field was investigated by Hayat and Kara [10], were they considered the fluid to be in an annular region between two coaxial cylinders. The axial couette flow problem of an electrically conducting fluid in an annulus was examined by Zaman et al. [11].
Unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer was studied by Joseph et al. [12]. The lower plate was considered porous. The governing equations of the flow field were solved by variable separable technique and the expression for the velocity, temperature, skin frictions and Nusselt numbers were obtained. Joseph et al. [15] examined the heat transfer on unsteady MHD flow of non-Newtonian fluid in horizontal parallel plates. The plates are arranged so that the upper plate oscillates and moves while the lower plate is stationary. In another paper by Joseph *et al.* [16], they investigated the unsteady MHD flow of a non-Newtonian

fluid (fourth-grade fluid) in a horizontal parallel plates channel. The upper plate is oscillating and moving while the bottom plate is stationary.

They employed He-Laplace scheme to solve the governing equations. Motivated by the above literatures, this paper presents an analysis on mass transfer on unsteady MHD flow of non-Newtonian fluid in horizontal channel with chemical reaction effects. We also employed He-Laplace method explicitly.

**Formulation of the Problem**

We consider the unsteady flow of an electrically conducting incompressible non-Newtonian fluid a horizontal channel as shown in figure 1 below. The fluid is subjected to a uniform transverse magnetic field $B_0$. We assume the bottom plate is fixed (stationary) and the top plates is moving with constant velocity, $u$.

The chemically reactive flow $U_0$ is heading $x$ – direction along infinite porous plate. Here, $U_0$ is the uniform velocity and $C_\infty$ is the concentration.

Under the above consideration, the equations that describe the physical circumstances are;

$$\frac{\partial u}{\partial t}=v\frac{\partial^2 u}{\partial y^2}+\frac{\alpha_1 v}{\rho}\frac{\partial^3 u}{\partial y^2\partial t}+\frac{\beta_1 v^2}{\rho}\frac{\partial^4 u}{\partial y^2\partial t^2}+\frac{6(\beta_2+\beta_3)}{\rho}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\frac{\gamma_1 v^3}{\rho}\frac{\partial^5 u}{\partial y^2\partial t^3}+\frac{2v(3\gamma_2+\gamma_3+\gamma_4+\gamma_5+\gamma_7+\gamma_8)}{\rho C_p}\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-\frac{\sigma B_0^2}{\rho C_p}u+g\beta_T(T-T_\infty)+g\beta_C(C-C_\infty)-\frac{v}{k}u \tag{1}$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial y^2}-K_c(C-C_\infty) \tag{2}$$

The initial and boundary conditions are

$$\left.\begin{array}{l} u(y,t)=e^{-yh}, C(y,t)=e^{-yh} \ \ at\ t=0\ for\ \ 0\le y\le h \\ u(y,t)=U, C(y,t)=C_w\ at\ y=h\ for\ t\ge 0 \\ u(y,t)\to\infty, C(y,t)\to\infty\ as\ y\to\infty\ for\ t>0 \end{array}\right\} \tag{3}$$

where $u$ is the fluid velocity, $C$ is the species concentration equation, $C_w$ is the concentration at the surface, is the density of the fluid, $C_p$ is the heat capacity, $B_0$ is the external magnetic field.

In order to transform equations (1) – (3), we use the following dimensionless parameters

$$u=\frac{u^*}{U_0}, p^*=\frac{p}{\mu h^2}, t=\frac{vt^*}{h^2}, G_c=\frac{g\beta_T(C_w^*-C_\infty^*)h^2}{v^2}, Ha^2=\frac{\sigma B_0^2 h^2}{\rho v}, Da=\frac{K}{h^2}, S_c=\frac{v}{D}, y=\frac{y^*}{h}, x=\frac{x^*}{h}, C=\frac{C^*-C_0^*}{C_w^*-C_\infty^*}, \tag{4}$$

Substituting equation (4) into equations (1) – (3) and by dropping the asterisks, we have the following:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+\alpha_1\frac{\partial^3 u}{\partial y^2\partial t}+\beta_1\frac{\partial^4 u}{\partial y^2\partial t^2}+l_1\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\gamma_1\frac{\partial^5 u}{\partial y^2\partial t^3}+l_2\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-l_3 u+G_c C \tag{5}$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}-K_r C \tag{6}$$

And the initial and boundary conditions become

$$\left.\begin{array}{l} u(y,t)=e^{-y}, C(y,t)=e^{-y}\ at\ t=0\ for\ 0\le y\le 1 \\ u(y,t)=1, C(y,t)=1\ at\ y=1\ for\ t\ge 0 \\ u(y,t)\to\infty, C(y,t)\to\infty\ \ as\ y\to\infty\ for\ t>0 \end{array}\right\} \tag{7}$$

where, $l_1=6(\beta_2+\beta_3), l_2=2(3\gamma_2+\gamma_4+\gamma_5+\gamma_7+\gamma_8), l_3=Ha^2+\frac{1}{Da}$ (7a)

**Method of Solution/Solution of the Problem**

In this section we employed the He – Laplace scheme to solve equations (6) and (7) subjects to the initial and boundary conditions (7).

Since equation (6) is a coupled non – linear partial differential equation, we have to solve equations (6) first.

Now applying Laplace transform on equation (6) we have,

$$L\left\{\frac{\partial C}{\partial t}\right\}=\frac{1}{S_c}L\left\{\frac{\partial^2 C}{\partial y^2}\right\}-L\{K_r C\} \tag{8}$$

Applying the initial condition and dividing through by $S$ and rearranging we get;

$$L\{C(y,t)\} = \frac{e^{-y}}{s} - \frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2 C}{\partial y^2}\right\} - L\{K_r C\}\right\} \quad (9)$$

Taking the inverse Laplace transform of both sides of equation (9), results in

$$C(y,t) = e^{-y} + L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2 C}{\partial y^2}\right\} - L\{K_r C\}\right\}\right] \quad (9)$$

Taking the inverse Laplace transform of both sides of equation (9), results in

$$C(y,t) = e^{-y} + L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2 C}{\partial y^2}\right\} - L\{K_r C\}\right\}\right] \quad (10)$$

Applying the Homotopy perturbation technique, equation (10) yields

$$\sum_{n=0}^{\infty} P^n C_n(y,t) = e^{-y} + P\left[L^{-1}\left\{\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2 C}{\partial y^2}\right\} - L\{K_r C\}\right\}\right\}\right] \quad (11)$$

Comparing the coefficients of the like powers of $'P'$ in equation (11) the following approximations were obtained;

$$P^0 : C_0(y,t) = e^{-y} \quad (12)$$

$$P^1 : C_1(y,t) = L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2 C_0}{\partial y^2}\right\} - L\{K_r C_0\}\right\}\right] = L^{-1}\left\{\frac{1}{S_c}\left(\frac{e^{-y}}{s^2}\right) - K_r\left(\frac{e^{-y}}{s^2}\right)\right\}$$

$$= \frac{e^{-y}}{S_c}t - K_r e^{-y} t \quad (13)$$

$$P^2 : C_2(y,t) = L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2 C_1}{\partial y^2}\right\} - L\{K_r C_1\}\right\}\right]$$

$$= L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{e^{-y}t}{S_c} - K_r e^{-y} t\right\} - L\left\{K_r\left(\frac{e^{-y}t}{S_c} - K_r e^{-y} t\right)\right\}\right\}\right]$$

$$= \frac{e^{-y}t^2}{2!S_c^2} - \frac{K_r e^{-y} t^2}{S_c} + \frac{K_r^2 e^{-y} t^2}{2!} \quad (14)$$

In viewing equations (12), (13) and (14), the solution to equation (6) is

$$C(y,t) = C_0(y,t) + C_1(y,t) + C_2(y,t) + \cdots$$

$$C(y,t) = e^{-y} + \left(\frac{e^{-y}}{S_c} - K_r e^{-y}\right)t + \left(\frac{e^{-y}}{S_c^2} + K_r^2 e^{-y} - \frac{2K_r e^{-y}}{S_c}\right)\frac{t^2}{2!} + \cdots \quad (15)$$

We now solve equation (5):

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \alpha_1 \frac{\partial^3 u}{\partial y^2 \partial t} + \beta_1 \frac{\partial^4 u}{\partial y^2 \partial t^2} + l_1\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} + \gamma_1 \frac{\partial^5 u}{\partial y^2 \partial t^3} + l_2\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^3 u}{\partial y^2 \partial t}\right] - l_3 u + G_c C$$

Applying the Laplace transform on both sides of equation (5) gives;

$$L\left\{\frac{\partial u}{\partial t}\right\} = L\left\{\frac{\partial^2 u}{\partial y^2} + \alpha_1 \frac{\partial^3 u}{\partial y^2 \partial t} + \beta_1 \frac{\partial^4 u}{\partial y^2 \partial t^2} + l_1\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} + \gamma_1 \frac{\partial^5 u}{\partial y^2 \partial t^3} + l_2\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^3 u}{\partial y^2 \partial t}\right] - l_3 u + G_c C\right\} \quad (16)$$

But,

$$L\left\{\frac{\partial u}{\partial t}\right\} = sL\{u(y,t)\} - u(y,0) \quad (17)$$

Hence,

$$L\{u(y,t)\} = \frac{u(y,0)}{s} + \frac{1}{s}L\left\{\frac{\partial^2 u}{\partial y^2} + \alpha_1 \frac{\partial^3 u}{\partial y^2 \partial t} + \beta_1 \frac{\partial^4 u}{\partial y^2 \partial t^2} + l_1\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} + \gamma_1 \frac{\partial^5 u}{\partial y^2 \partial t^3} + l_2\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \right.\right.$$

$$\left.\left.\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^3 u}{\partial y^2 \partial t}\right] - l_3 u + \frac{G_c}{s}\left(e^{-y} + \frac{e^{-y}}{S_c}t - K_r e^{-y} t + \frac{e^{-y}t^2}{2!S_c^2} - \frac{K_r e^{-y} t^2}{S_c} + \frac{K_r^2 e^{-y} t^2}{2!}\right)\right\} \quad (18)$$

Taking the inverse Laplace transform of both sides of equation (18), we have;

$$L^{-1}\{L\{u(y,t)\}\} = L^{-1}\left\{\frac{u(y,0)}{s} + \frac{1}{s}L\left\{\frac{\partial^2 u}{\partial y^2} + \alpha_1 \frac{\partial^3 u}{\partial y^2 \partial t} + \beta_1 \frac{\partial^4 u}{\partial y^2 \partial t^2} + l_1\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} + \gamma_1 \frac{\partial^5 u}{\partial y^2 \partial t^3} + \right.\right.$$

$$\left.\left. l_2\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^3 u}{\partial y^2 \partial t}\right] - l_3 u + \frac{G_c}{s}\left(e^{-y} + \frac{e^{-y}}{S_c}t - K_r e^{-y} t + \frac{e^{-y}t^2}{2!S_c^2} - \frac{K_r e^{-y} t^2}{S_c} + \frac{K_r^2 e^{-y} t^2}{2!}\right)\right\}\right\} \quad (19)$$

Or,

$$u(y,t) = e^{-y} + l_4 e^{-y}t + G_c e^{-y}t + G_C e^{-y}t + \frac{G_c e^{-y}t^2}{2!S_c} - \frac{G_c K_r e^{-y}t^2}{2!} + \frac{G_c e^{-y}t^3}{3!S_c^2} - \frac{2G_c K_r e^{-y}t^3}{3!} + \frac{2G_c K_r^2 e^{-y}t^3}{3!} + L^{-1}\left\{\frac{1}{s} L\left\{\frac{\partial^2 u}{\partial y^2} + \alpha_1 \frac{\partial^3 u}{\partial y^2 \partial t} + \beta_1 \frac{\partial^4 u}{\partial y^2 \partial t^2} + l_1 \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} + \gamma_1 \frac{\partial^5 u}{\partial y^2 \partial t^3} + l_2\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^3 u}{\partial y^2 \partial t}\right] - l_3 u\right\}\right\} \tag{20}$$

Applying the Homotopy perturbation method to equation (20), gives,

$$\sum_{n=0}^{\infty} P^n u_n(y,t) = e^{-y} + l_4 e^{-y}t + G_c e^{-y}t + G_C e^{-y}t + \frac{G_c e^{-y}t^2}{2!S_c} - \frac{G_c K_r e^{-y}t^2}{2!} + \frac{G_c e^{-y}t^3}{3!S_c^2} - \frac{2G_c K_r e^{-y}t^3}{3!} + \frac{2G_c K_r^2 e^{-y}t^3}{3!} + P\left(L^{-1}\left\{\frac{1}{s} L\left\{\frac{\partial^2 u}{\partial y^2} + \alpha_1 \frac{\partial^3 u}{\partial y^2 \partial t} + \beta_1 \frac{\partial^4 u}{\partial y^2 \partial t^2} + l_1 H_a(u_n) + \gamma_1 \frac{\partial^5 u}{\partial y^2 \partial t^3} + l_2\left[2H_b(u_n) + H_c(u_n)\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^3 u}{\partial y^2 \partial t}\right] - l_3 u\right\}\right\}\right) \tag{21}$$

Where, $H_a(u_n), H_b(u_n)$ and $H_c(u_n)$ are the He's polynomials for $\left(\frac{\partial u}{\partial y}\right)^2, \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y}$ and $\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^3 u}{\partial y^2 \partial t}$

respectively. Now, comparing the like powers of "$P$" in equation (21) and equating their coefficients gives

$$P^0; u_0(y,t) = e^{-y} + l_4 e^{-y}t + G_c e^{-y}t + G_C e^{-y}t + \frac{G_c e^{-y}t^2}{2!S_c} - \frac{G_c K_r e^{-y}t^2}{2!} + \frac{G_c e^{-y}t^3}{3!S_c^2} - \frac{2G_c K_r e^{-y}t^3}{3!} + \frac{2G_c K_r^2 e^{-y}t^3}{3!} \tag{22}$$

$$P^1; u_1(y,t) = L^{-1}\left\{\frac{1}{s} L\left\{\frac{\partial^2 u}{\partial y^2} + \alpha_1 \frac{\partial^3 u}{\partial y^2 \partial t} + \beta_1 \frac{\partial^4 u}{\partial y^2 \partial t^2} + l_1 (u_0')^2 + \gamma_1 \frac{\partial^5 u}{\partial y^2 \partial t^3} + l_2[2u_0''' u''_{0t} + (u_0')^2 (u_0'' u'_{0t})] - l_3 u\right\}\right\} \tag{23}$$

And,

$$\begin{aligned}
u_1(y,t) = {} & \Big(e^{-y} + \alpha_1 G_c e^{-y} - \beta_1 G_c K_r e^{-y} - 2\gamma_1 G_c K_r e^{-y} + \gamma_1 G_c K_r^2 e^{-y} + 2\gamma_1 \delta G_r e^{-y} + \\
& L_3 e^{-y} + \frac{\gamma_1 G_c e^{-y}}{S_c} + 2L_2 G_c e^{-2y} + L_1 e^{-2y}\Big)t + \Big(G_c e^{-y} - \alpha_1 G_c K_r e^{-y} - 2\beta_1 G_c K_r e^{-y} + \\
& \beta_1 G_c K_r^2 e^{-y} + \frac{\alpha_1 G_c e^{-y}}{S_c} + \frac{\beta_1 G_c e^{-y}}{S_c^2} + L_3 L_4 + L_3 G_c e^{-y} + L_3 G_r e^{-y} + 2L_1 G_c e^{-2y} - 2L_2 G_c K_r e^{-2y} + \\
& 2L_2 G_c^2 e^{-2y} + \frac{2L_2 G_c e^{-2y}}{S_c}\Big)\frac{t^2}{2!} + \Big(\alpha_1 G_c K_r^2 e^{-y} - G_c K_r e^{-y} - 2\alpha_1 G_c K_r e^{-y} + \frac{G_c e^{-y}}{S_c} + \frac{\alpha_1 G_c e^{-y}}{S_c^2} - \\
& 2L_1 G_c K_r e^{-2y} + 2L_1 G_c^2 e^{-2y} + \frac{2L_1 G_c e^{-2y}}{S_c} + L_3 e^{-y} - L_3 G_c K_r e^{-y} L_2 G_c K_r^2 e^{-2y} - 2L_2 G_c K_r^2 e^{-2y} + \\
& \frac{L_2 G_c e^{-2y}}{S_c^2} + \frac{2L_2 G_c^2 e^{-2y}}{S_c} + 2L_2 G_c K_r e^{-2y} + L_2 G_c^2 e^{-2y} - L_2 G_c^2 K_r e^{-2y}\Big)\frac{t^3}{3!} + \Big(2G_c K_r e^{-y} + \frac{G_c e^{-y}}{S_c} - \\
& 2L_1 G_c K_r e^{-2y} + 2L_1 G_c K_r^2 e^{-2y} - 6L_1 G_c^2 K_r e^{-2y} + \frac{2L_1 G_c e^{-2y}}{S_c^2} + \frac{4L_1 G_c^2 e^{-2y}}{S_c} + L_3 G_c K_r^2 e^{-y} - \\
& L_3 G_c K_r e^{-y} + \frac{L_3 G_c e^{-y}}{S_c^2} + \frac{L_2 G_c^2 e^{-2y}}{S_c^2} - 2L_2 G_c^2 K_r e^{-2y} + \frac{L_2 G_c^2 e^{-2y}}{S_c} - L_2 G_c^2 K_r e^{-2y} + \frac{4L_2 G_c^2 e^{-2y}}{3S_c^2} - \\
& \frac{4L_2 G_c^2 K_r e^{-2y}}{6} + \frac{2L_2 G_c^2 K_r^2 e^{-2y}}{6} + \frac{L_2 G_c^2 e^{-2y}}{2S_c^2} - L_2 G_c^2 K_r e^{-2y} + \frac{L_2 G_c^2 K_r^2 e^{-2y}}{2}\Big)\frac{t^4}{4!} + \Big(8L_1 G_c^2 e^{-2y} - \\
& 16L_1 G_c^2 K_r e^{-2y} + 14L_1 G_c^2 K_r^2 e^{-2y} + \frac{6L_1 G_c^2 e^{-2y}}{S_c^2} - \frac{12L_1 G_c^2 K_r e^{-2y}}{S_c} - \frac{12L_2 G_c^2 K_r e^{-2y}}{S_c^2} + \\
& 24L_2 G_c^2 K_r^2 e^{-2y} + \frac{24L_2 G_c^2 K_r^3 e^{-2y}}{2} + \frac{24L_2 G_c^2 e^{-2y}}{3S_c^3} - \frac{24L_2 G_c^2 K_r e^{-2y}}{3S_c^2} + \frac{24L_2 G_c^2 K_r e^{-2y}}{3S_c} - \frac{24L_2 G_c^2 K_r^3 e^{-2y}}{3}\Big)\frac{t^5}{5!} + \\
& \Big(20L_1 G_c^2 K_r^2 e^{-2y} - 20L_1 G_c^2 K_r^3 e^{-2y} + \frac{20L_1 G_c^2 e^{-2y}}{S_c^3} - \frac{40L_1 G_c^2 K_r e^{-2y}}{S_c} + \frac{120L_2 G_c^2 e^{-2y}}{6S_c^4} - \\
& \frac{120L_2 G_c^2 K_r e^{-2y}}{3S_c^2} + \frac{120L_2 G_c^2 K_r^2 e^{-2y}}{6S_c^2} - \frac{120L_2 G_c^2 K_r e^{-2y}}{3S_c^2} - \frac{120L_2 G_c^2 K_r^2 e^{-2y}}{3S_c^2} + \frac{120L_2 G_c^2 K_r^2 e^{-2y}}{6S_c^2} - \\
& \frac{120L_2 G_c^2 K_r^3 e^{-2y}}{3} + \frac{120L_2 G_c^2 K_r^4 e^{-2y}}{6}\Big)\frac{t^6}{6!} + \Big(80L_1 G_c^2 K_r^2 e^{-2y} - 80L_1 G_c^2 K_r^3 e^{-2y} + 20L_1 G_c^2 K_r^4 e^{-2y} + \\
& \frac{20L_1 G_c^2 e^{-2y}}{S_c^4} - \frac{80L_1 G_c^2 K_r e^{-2y}}{S_c^2} + \frac{40L_1 G_c^2 K_r^2 e^{-2y}}{S_c^2}\Big)\frac{t^7}{7!} + \cdots
\end{aligned} \tag{24}$$

Therefore, the solution to equation (5) is

$$u(y,t) = u_0(y,t) + u_1(y,t) + \cdots \tag{25}$$

Where, $u_0(y,t)$ and $u_1(y,t)$ are defined in equations (22) and (24) respectively.

**Results and Discussion**

The study has been analyzed on chemically reactive convective non-Newtonian fluid in a horizontal channel.

The influence of chemical reaction parameter along with diversified physical parameters are depicted graphically on different flow fields using MATLAB. The default values for the pertinent parameters are taken as.

$$\alpha = 0.3, \beta_1 = 1, \beta_2 = 1, \beta_3 = 1, \gamma_1 = 0.3, \gamma_2 = 0.3, \gamma_4 = 0.3, \gamma_5 = 0.3, \gamma_7 = 0.3, \gamma_8 = 0.3, S_c = 1,\ G_c = 5, Ha = 1, Da = 1, K_r = 1, t = 0.5.$$

Figures 1 and 2 show the effect of chemical reaction parameter on velocity profile and concentration distribution respectively. It can be seen that the chemical reaction parameter decreases both the flow fields. Increase in the value of chemical parameter implies more interaction of species concentration with the momentum boundary layer and less interaction with thermal boundary layer.

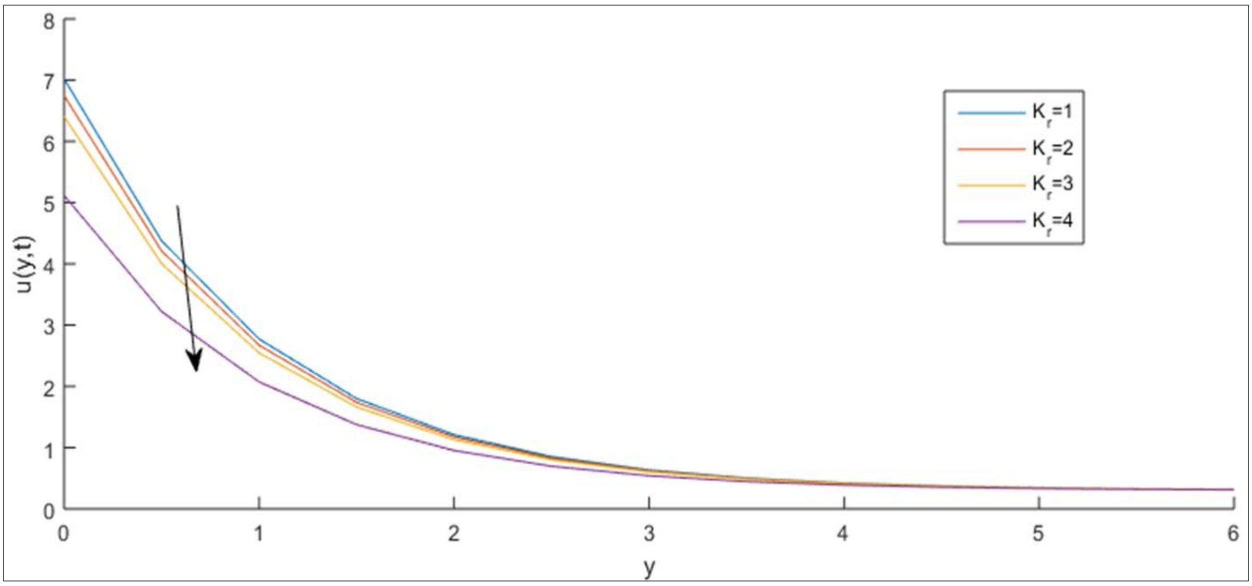


**Fig 1:** Effect of chemical reaction parameter $K_r$ on velocity profile $u$

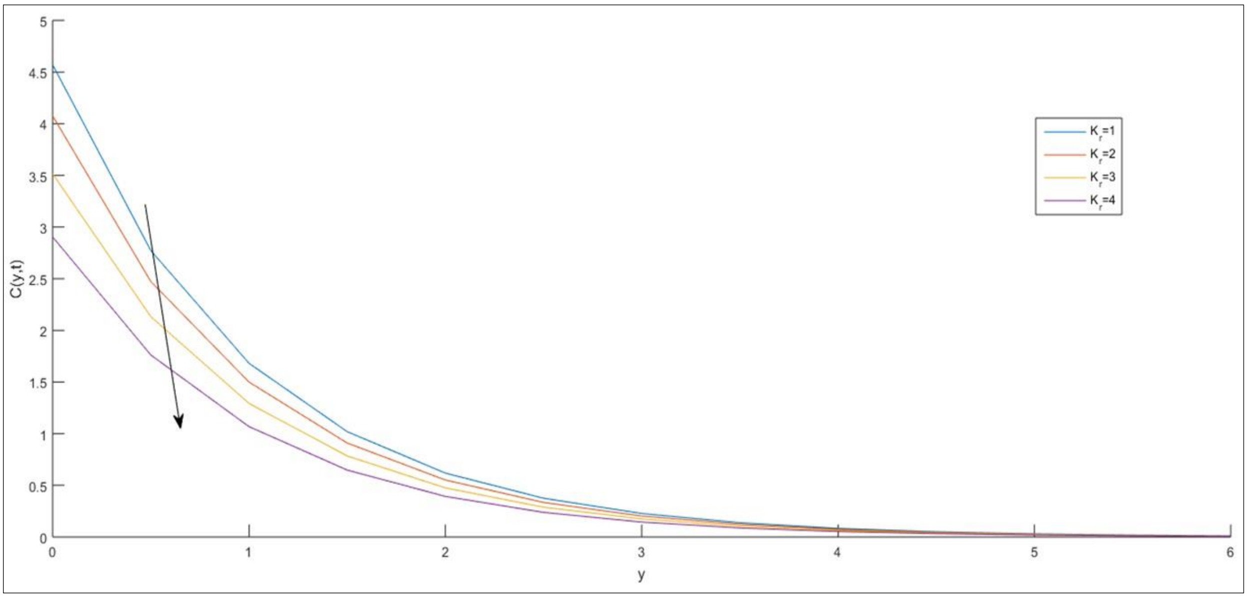


**Fig 2:** Effect of Chemical reaction parameter $K_r$ on Concentration distribution $C$

The effect of the Hartman number $Ha$ i.e the magnetic field intensity on velocity profile is depicted in figure 3. It is interesting to know that increase in magnetic field intensity decreases the magnitude of fluid velocity. To this effect the magnetic field intensity suppresses the turbulence of the flow. Physically, when magnetic field is applied to any fluid, the apparent viscosity of the fluid increases to the point of becoming visco elastic solid. It is of great interest that, yield stress of the fluid can be controlled accurately through variation of magnetic field intensity. The result is that the ability of the fluid to transmit force can be controlled with the help of electromagnet which give rise to many possible controlled-based applications, including electromagnetic casting of metals, ion propulsion etc

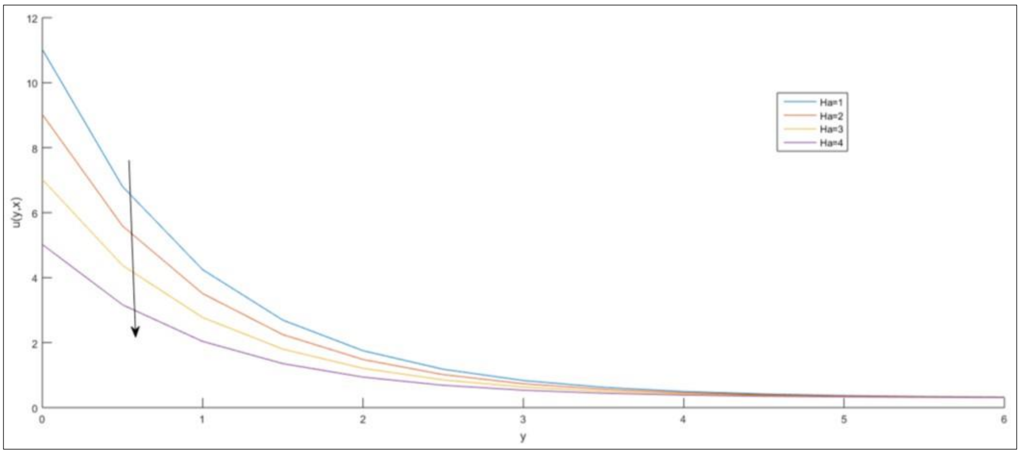


**Fig 3:** Effect of Hartmann number on velocity profile $u$

Figure 4 illustrates the effect of Grashof number $G_c$ due to mass transfer on velocity profile. The Grashof number provides the relation between inertia force, buoyant force and viscous force. It can be seen that increase in Grashof number enhances the velocity significantly. To this effect, at higher Grashof number, the flow at the boundary is turbulent while, at lower Grashof number the flow at the boundary is laminar.

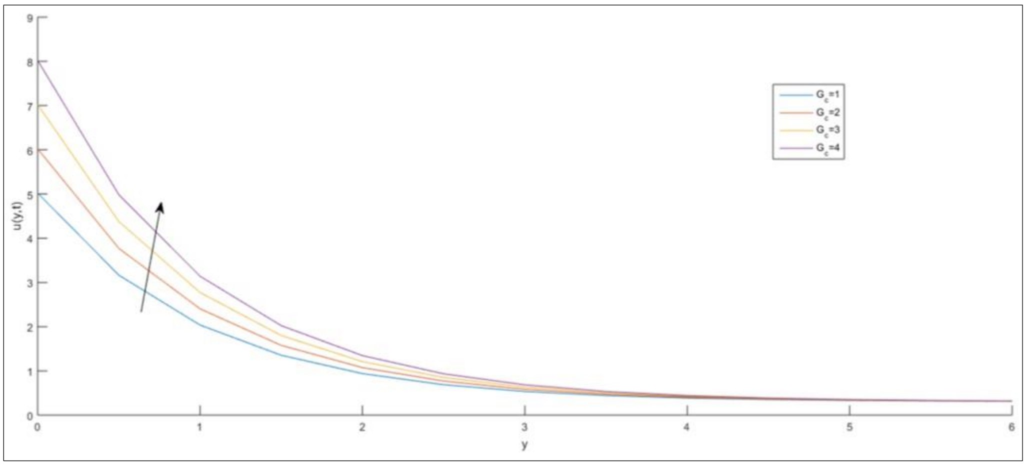


**Fig 4:** Effect of Grashof number $G_c$ on velocity profile $u$

Figures 5 and 6 depict the effect of Schmidt number on velocity profile and concentration distribution. The Schmidt number is defined as the ratio of momentum diffusivity (kinematic viscosity) and mass diffusivity. It is observed that the increase in Schmidt number decreases both the velocity profile and concentration distribution. Physically, Schmidt number $(S_c)$ helps to develop fluid concentration and concentration buoyancy force. Furthermore, it can also be used to improve the visualization of fluid fields.

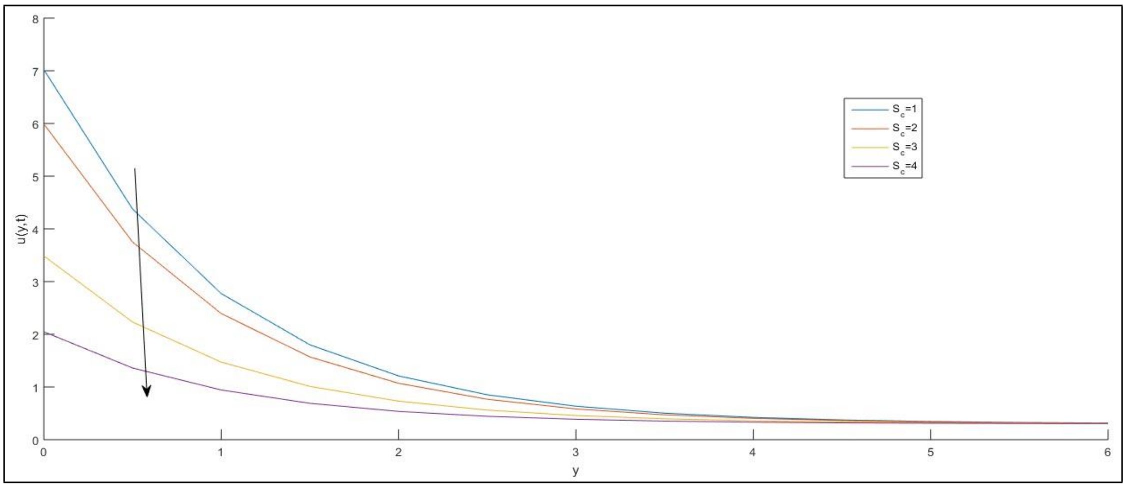


**Fig 5:** Effect of Schmidt number on velocity profile $u$

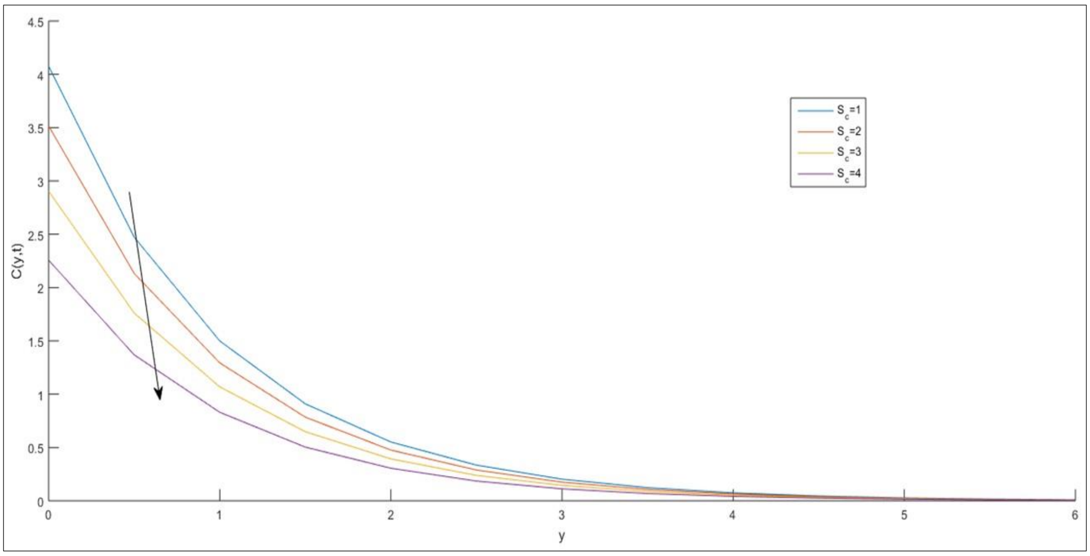


**Fig 6:** Effect of Schmidt number $S_c$ on Concentration distribution $C$

## Conclusion

The mass transfer on unsteady MHD flow of non-Newtonian fluid in horizontal channel with chemical reaction effects have been considered. The equations that govern the flow are transformed into non-dimensional form using dimensionless parameters. The non-dimentional equations are solved by implementing He-Laplace method. The key findings are given below;

For upsurging data of chemical reaction, velocity and concentration fields diminish.

Velocity decline due to the increment of magnetic parameter.

Strong values of Schmidt number decreases both the velocity and the concentration fields the boundary layer of the Sherwood number field.

Increase in Grashof number accelerate the velocity field.

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